Opinionator
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Paradoxical Truth

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The Stone is a forum for contemporary philosophers on issues both timely and timeless.

Professor Greene is lecturing. Down the hall, her arch-rival, Professor Browne, is also lecturing. Professor Greene is holding forth at length about how absurd Professor Browne’s ideas are. She believes Professor Browne to be lecturing in Room 33. So to emphasize her point, she writes on the blackboard the single sentence:

Everything written on the board in Room 33 is false.

But Professor Greene has made a mistake. She, herself, is in Room 33. So is what she has written on the board true or false? If it’s true, then since it itself is written on the board, it’s false. If it’s false, then since it is the only thing written on the board, it’s true. Either way, it’s both true and false.

Philosophers and logicians love paradoxes, and this is one — one of the many versions of what is usually called the Liar Paradox, discovered by the ancient Greek philosopher Eubulides (4th century B.C.).

Paradoxes are apparently good arguments that lead to conclusions that are beyond belief (Greek: “para” = beyond, “doxa” = belief). And when you meet a paradox, you’ve got only two choices. One is to accept that the conclusion, implausible as it may seem, is actually true; the other is to reject the conclusion, and explain what has gone wrong in the argument.

Both responses are possible. To illustrate the first, here’s another paradox. The whole numbers and the even whole numbers can be paired off, one against the other, as follows:

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & \ldots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \ldots \\ 2 & 4 & 6 & 8 & \ldots \end{array}$$

This appears to show that there are exactly the same number of even numbers as whole numbers. That seems false, since obviously the even numbers leave some numbers out.

This paradox was known to the medievals, and to Galileo. So let's call it Galileo's Paradox. Until the 19th century, the paradox was taken to show that the whole notion of infinity was incoherent. But towards the end of that century, the work of the German mathematician Georg Cantor on the infinite led to one of the most major revolutions in the history of mathematics. Fundamental to it was accepting that there are indeed exactly as many even numbers as whole numbers. It is the very *nature* of infinite totalities that you can throw away some of their members, and have as many as you started with.

The other possibility (saying what is wrong with the argument) is illustrated by another paradox. Another Ancient Greek philosopher, Zeno, who flourished about a century before Eubulides, produced a number of paradoxes concerning motion. Here's one of them, often called the *Dichotomy*. Suppose a car is moving from to *A* to *B* . Let's measure the distance between *A* and *B* by a scale according to which *A* is at point 0 and *B* is at point 1. Then before the car gets to point 1, it has to get half way there, point 1/2; and once it has got there, it has to get to a point half way between 1/2 and 1, 3/4; and so on… In other words, it has to get to every one of the infinite number of points 1/2, 3/4, 7/8, … But you can't do an infinite number of things in a finite time. So the car never get to point *B*.

In wrestling with the Liar Paradox for 2500 years, maybe we have been trying to find a fault where there is none.

Here we can't just accept the conclusion: we know that the car can get to point *B*. So something must be wrong with the argument. In fact, there is now a general consensus about what is wrong with it (based on other developments in 19th-century mathematics concerning infinite series). You *can* do an infinite number of things in a finite time — at least provided that these things can be done faster and faster.

So let's come to back to the Liar Paradox. Which of the two kinds of paradox is this? Can we accept the conclusion, or must there be something wrong with the argument? Well, notice that the conclusion of the argument is a bald contradiction: the claim on the blackboard is both true and false. Now, the principle of noncontradiction says that you can never accept a contradiction.

And the principle of noncontradiction has been high orthodoxy in Western philosophy since Aristotle mounted a spirited defense of it in his "Metaphysics" — so orthodox that no one seems to have felt the need to mount a sustained defense of it ever since. So the paradox must be of the second kind: there must be something wrong with the argument. Or must there?

Not according to a contentious new theory that's currently doing the rounds. According to this theory, some contradictions are actually true, and the conclusion of the Liar Paradox is a paradigm example of one such contradiction. The theory calls a true contradiction a *dialetheia* (Greek: "di" = two (way); "aletheia" = truth), and the view itself is called dialetheism. One thing that drives the view is that cogent diagnoses of what is wrong with the Liar argument are seemingly impossible to find. Suppose you say, for example, that paradoxical sentences of this kind are simply meaningless (or neither true nor false, or some such). Then what if Professor Greene had written on the board:

Everything written on the board in Room 33 is either false or meaningless.

If this were true or false, we would be in the same bind as before. And if it's meaningless, then it's *either* false *or* meaningless, so it's true. We are back with a contradiction. This sort of situation (often called a *strengthened paradox*) affects virtually all suggested attempts to explain what has gone wrong with the reasoning in the Liar Paradox.

At any rate, even after two and a half thousand years of trying to come up with an explanation of what is wrong with the argument in the Liar Paradox, there is still no consensus on the matter. Contrast this with Zeno's paradoxes, where there is virtually complete consensus. Maybe, then, we have just been trying to find a fault where there is none.

Of course, this means junking the principle of noncontradiction. But why should we accept that anyway? You might think that since Aristotle's defense established the principle in Western philosophy, his arguments must have been pretty good. Were they? No. The main argument is so tortured that experts can't even agree on *how* it is meant to work, let alone *that* it works. There's a bunch of smaller arguments as well, but most of these are little more than throw-away comments, many of which are clearly beside the point. Interestingly, virtually everything else that Aristotle ever defended has been overthrown — or at least seriously challenged. The principle of noncontradiction is, it would seem, the last bastion!

Naturally, there is more to be said about the matter — as there always is in philosophy. If you ask most modern logicians why there can be no true contradictions, they will probably tell you that everything follows logically from a contradiction, so if even one contradiction were true, everything would be true. Clearly, everything is too much!

This principle of inference that everything follows from a contradiction sometimes goes by its medieval name, *ex falso quodlibet*, but it is often now called by a more colorful name: explosion. There is, in fact, a connection between explosion and the principle of noncontradiction. A common suggestion of what it is for *B* to follow logically from *A* is that you can't have *A* without having *B*. Given the principle of noncontradiction, if *A* is a contradiction, you can't have it. And if you can't have *A*, you certainly can't have *A and B*. That is, everything follows from a contradiction.

Evidently, if this argument is invoked against dialetheism, it is entirely question-begging, since it takes for granted the principle of noncontradiction, which is the very point at issue.

Moreover, for all its current orthodoxy, explosion seems a pretty implausible principle of inference. It tells us, after all, that if, for example, Melbourne were and were not the capital of Australia, Caesar would have invaded England in 1066. There really doesn't seem to be much connection between these things. Explosion would itself seem to be a pretty paradoxical consequence of whatever it is supposed to follow from.

Unsurprisingly, then, the last 40 years or so have seen fairly intensive investigations of logics according to which explosion is not correct. These are called paraconsistent logics, and there is now a very robust theory of such logics. In fact, the mathematical details of these logics are absolutely essential in articulating dialetheism in any but a relatively superficial way. But the details are, perhaps, best left for consenting logicians behind closed doors.

You might think that there is another problem for dialetheism: if we could accept some contradictions, then we could never criticize someone whose views were inconsistent, since they might just be true. Suppose that I am charged with a crime. In court, I produce a cast-iron alibi, showing that I was somewhere else. The prosecutor accepts that I was not at the crime scene, but claims that I was there anyway. We certainly want to be able to say that this is not very sensible!

But the fact that it is rational to accept *some* contradictions does not mean that it is rational to accept *any* contradiction. If the principle of noncontradiction fails, then contradictions cannot be ruled out by logic alone. But many things cannot be ruled out by logic alone, though it would be quite irrational to believe them. The claim that the earth is flat is entirely consistent with the laws of logic. It's crazy for all that.

And no one has yet mastered the trick of being in two places at the same time, as both we and the prosecutor know.

Indeed, if you consider all the statements you have met in the last 24 hours (including the ones in this article), the number that might plausibly be thought to be dialetheias is pretty small. So it seems safe to assume that the probability of any given contradiction being true is pretty low. We have, then, quite good *general* grounds for rejecting a contradiction we come across. But of course, those general grounds may be trumped on the occasions where we do have good reason to believe that the contradiction is true — as with the Liar Paradox.

If dialetheias are pretty rare, and if they appear to be fairly esoteric things like the Liar sentence, you might wonder why we should bother about them at all. Why not just ignore them? One ignores them at great risk. Scientific advances are often triggered by taking oddities seriously. For example, at the end of the 19th century, most physicists thought that their subject was pretty much sewn up, except for a few oddities that no one could account for, such as the phenomenon of black-body radiation. Consideration of this eventually generated quantum theory. Had it been ignored, we would not have had the revolution in physics produced by the theory. Similarly, if Cantor had not taken Galileo’s paradox seriously, one of the most important revolutions in mathematics would never have happened either.

Revolutions in logic (of various kinds) have certainly occurred in the past. Arguably, the greatest of these was around the turn of the 20th century, when traditional Aristotelian logic was overthrown, and the mathematical techniques of contemporary logic were ushered in. Perhaps we are on the brink of another.
